PiXL Independence:

Mathematics - Student Booklet

KS4 HIGHER

Topic 2 – Powers and Roots, HCF/LCM. Ratio and Proportion.

Contents:

I. Basic Skills Check

Answer the following questions. In order to improve your basic arithmetic you should attempt these without a calculator where possible.

Skills Check 1

1. A phone is reduced by 15% to a price of £140.25. Calculate the original cost of the system.

2. What is $\frac{2}{3}$ of £540?

3. Write 26 as a product of its prime factors.

4. Factorise $x^2 - 2x - 24$.

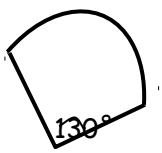
5. Write 620,000,000 in standard form.

6. Solve: $2x + 7 \leq -11.$

7. Use prime factors to find the lowest common multiple of 170 and 80.

8. List the first 5 terms of the sequence 4n- 2.

9. Find the total perimeter of the sector shown, correct to one decimal place.



4cm

10. Calculate $(4.25 \times 10^{15}) \div (1.8 \times 10^{9})$, giving your answer in standard form correct to **two** significant figures.

Skills Check 2

1. A year ago Peter was 152 cm. He is now 4% taller. How tall is Peter now?

2. Calculate $(2.1\times10^{12})\times(5.2\times10^{3})$, giving your answer in standard form correct to two significant figures.

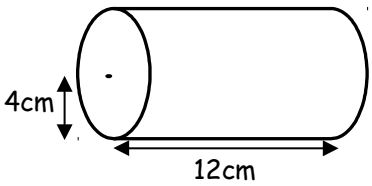
3. Write 48 as a product of prime factors. What is the LCM of 48 and 54?

4. Factorise x^2 -20x + 96.

5. Write 0.000000754 in standard form.

6. Solve: $-19\leq6-5x$.

7. Calculate the total surface area of a cylinder with radius 4cm and length 12cm. Give your **final** answer to one decimal place.



8. Find the nth term of the sequence: 4, 7, 12, 19,....

9. Find the reciprocal of the number 2.8, giving your answer as a fraction.

10. A class of 14 boys and 18 girls take a test. The mean mark for the boys is 65%., and the mean mark for the girls is 80%. Calculate the mean mark for the whole class.

Skills Check 3

1. A shopkeeper buys small scooters at £45. She sells the bikes making 15% profit. Calculate the selling price of the scooters.

2. George uses ¾ of a tin of dog food each day. How many tins does he use in 7 days?

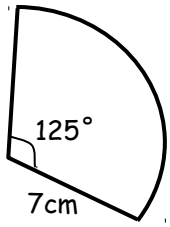
3. Find the HCF of 48 and 16.

4. Factorise $x^2 + x - 30$.

5. Write 0.0000000758 in standard form.

6. Solve: $5x + 7 \geq 3x - 2$.

7. Find the total perimeter of the sector shown, correct to one decimal place.



8. Find the nth term of the sequence: -1, 2, 7, 14.

9. Use prime factors to find the highest common factor of 250 and 904.

10. A class of 12 boys and 18 girls take a test. The mean mark for the class is 75%. If the mean mark for the boys is 64%, find the mean mark for the girls.

II. Short Exam Questions

Section 1 – Index Laws

1. Simplify each of these expressions, giving each answer in the form y^n:
   a. $y^3 \times y^5$
   b. $\sqrt[3]{y}$
   c. $\frac{1}{y^2}$
   d. $\left(y^{-4}\right)^3$
   e. $\frac{y^{12}}{y^3}$
   f. $\frac{y^5 \times y}{y^4}$

2. Evaluate each of the following:
   a. $(27)^{\frac{1}{3}}$
   b. $(12)^{-2}$
   c. 5^0
   d. $(27)^{\frac{4}{3}}$
   e. $(49)^{-\frac{1}{2}}$

3. Write each of the following as powers of 8:
   a. $\frac{1}{8}$
   b. 2
   c. 1
   d. $\frac{1}{64}$

4. Solve these exponential equations.
   a. $2^x = 2$
   b. $36^x = 6$
   c. $7^x = \frac{1}{49}$

5. Expand the brackets and simplify the following expressions as far as possible:

   a. $(\sqrt{3}+5)(\sqrt{3}+1)$

   b. $(\sqrt{7}-1)^2$

   c. $(4+\sqrt{7})(4-\sqrt{7})$

6. Simplify these expressions as far as possible by first 'rationalising the denominator' in each case:

   a. $\frac{12}{\sqrt{3}}$

   b. $\frac{30}{\sqrt{6}}$

   c. $\frac{14}{(3+\sqrt{2})}$

7. Expand the brackets and simplify the following expressions as far as possible:

   a. $(\sqrt{3}+5)(\sqrt{3}+1)$

   b. $(\sqrt{7}-1)^2$

   c. $(4+\sqrt{7})(4-\sqrt{7})$

   d. $(1+3\sqrt{2})(5-2\sqrt{2})$

Section 2 - HCF/LCM

1. Drummers hit their drums on certain beats. Drummer A hits his drum every 6 seconds. Drummer B hits his drum every 7 seconds. In the first 60 seconds, will they ever play at the same time?

2. Are these correct? If not correct them.
   a) HCF of 21 and 28 is 7. b) HCF of 27 and 45 is 5. c) HCF of 28 and 16 is 8.

3. a) Write 48 and 120 in prime factor form.
   b) Use your answers to find their highest common factor.

4. As a product of prime factors $60 = 2^2 \times 3 \times 5$.
   a) What number is represented by $2 \times 3^2 \times 5$?
   b) Find the lowest common multiple of 60 and 48.

5. Tom, Sam and Matt are counting drum beats.
   Tom hits a snare drum every 2 beats.
   Sam hits a kettle drum every 5 beats.
   Matt hits a bass drum every 8 beats.

   Will they ever play at the same time during 60 beats?

   Show clearly how you get your answer.

6. "I am thinking of two numbers, the HCF of the two numbers is 2. The LCM is 210" what could my numbers be?

7. Using prime decomposition find the LCM and HCF for 240 and 36.

8. Hotdogs come in jars of 8 and cost £1.20 per jar. Bread rolls are in packs of 12 and cost 99p. How many do I need to buy so I have exactly the same number of hotdogs and bread buns and how much will that cost?

Section3 - Ratio and Proportion

1. V is inversely proportional to h. It is known that V = 18 when h = 24.
    a. Find the equation that connects V and h.
    b. Find V when h = 15.
    c. Work out h when V = 30.

2. T is directly proportional to r^3. It is known that T = 36 when r = 2.
    a. Find the equation that connects T and r.
    b. Find T when r = 6.
    c. Work out r when T = 7776.

3. On a map, the distance between two cities is 15cm. In real-life the distance is known to be 75km.

    Find the scale of the map, giving your answer in the form 1 : n.

4. In a school, the ratio of boys to girls is 4 : 5. If there are 424 boys in the school, how many girls are there?

5. Graham and Harry agree to share their weekly earnings of £748 in the ratio 3 : 5. How much will each of them receive?

6. Simplify each of these ratios as far as possible.
    a. 2kg : 800g.
    b. 45 minutes : 2 hours.
    c. 800mm : 40cm : 2metres.

7. Write the ratio 50 : 120 in the form 1 : n.

8. Write 240 : 600 in the form n : 1.

9. On a map, the distance between two towns is 12cm. In real-life the distance is known to be 18km. Find the scale of the map, giving your answer in the form 1 : n.

10. The cost of a certain type of carpet roll is directly proportional to the length bought. Sheila pays £17.55 for 3 metres of this carpet.
    a. Write a formula to give the **cost** in terms of the **length** of carpet.
    b. If Wayne buys 8 metres of the same carpet, how much will it cost?

11. P is directly proportional to t^2. It is known that P = 20.8 when t = 4.
    a. Find the proportionality equation that connects P and t.
    b. Use your equation to find P when t = 9.
    c. Find the value of t when P = 520.

12. R is inversely proportional to d. When d = 36, R = 4.
    a. Find the proportionality equation connecting R and d.
    b. Use this equation to find R when d = 10.
    c. Calculate d when R = 1.5.

Section4 - Mixed Questions

1. Three litres of diesel costs £2.82. What is the cost of 40 litres?

2. 300 grams of sweets cost £1.65. Find the cost of;

a) 100 grams of sweets
b) 500 grams of sweets
c) 5 kg of sweets.
Why might your answer to (c) be unrealistic?

3. a) Elana is paid £12.50 per hour. She is given a pay rise of 20%. What is her new pay rate?
b) Six months later, due to adverse economic conditions Elana is asked to take a 20% pay *cut*. What will be her hourly rate now?

4. Explain why 0.28 must be a rational number.

5. Without a calculator evaluate each of these;

a. $64^{\frac{2}{3}}$ b. 5^{-2} c. 18^{0}

6. Simplify as far as possible;

a. $\sqrt{18}+\sqrt{50}$ b. $(3+\sqrt{7})(4-\sqrt{7})$

7. Simplify these by first rationalising the denominator.

a. $\frac{12}{\sqrt{6}}$ b. $\frac{24}{(3-\sqrt{3})}$

8. A rectangle has an area of 8cm^2. If its width is $(3-\sqrt{7})$ cm, calculate its perimeter, giving your answer in the form $(A+B\sqrt{7})$ cm where *A* and *B* are integers.

9. On Saturday the ratio of adults to children in a swimming pool is 2:3. The swimmers are either in the main pool or on the slides. $\frac{1}{3}$ of the children are on the slides. 68 children are in the main pool.

How many people went swimming in total?

III. Further Practice

1. Match the correct people using the information given. Check your answer at the end.
https://justmaths.co.uk/Worksheets/Number/Factors%20multiples%20and%20primes%20-%20WORKSHEET.pdf
https://justmaths.co.uk/Worksheets/Number/Factors%20multiples%20and%20primes%20-%20ANSWERS.pdf

2. **Read the information carefully.** Has the bill been calculated correctly? If not what should it be? What mistakes have they made? You should write a letter of complaint to the garage explaining what they have mis-calculated.
https://justmaths.co.uk/Worksheets/Number/Functional%20Skills%20-%20GARAGE%20PRICING%20TASK.pdf

3. **Exam style practice.** For each of the topics you should watch the video, then answer the exam questions and mark your answers. Where have you made mistakes? Is there something you need to do more work on?

   Ratio and Proportion.
   http://www.mathsgenie.co.uk/direct-and-inverse-proportion.html
   http://www.mathsgenie.co.uk/resources/96_direct-and-inverse-proportion.pdf
   http://www.mathsgenie.co.uk/resources/96_direct-and-inverse-proportionans.pdf

   Fractional and negative indices.
   http://www.mathsgenie.co.uk/indices2.html
   http://www.mathsgenie.co.uk/resources/94_fractional-and-negative-indices.pdf
   http://www.mathsgenie.co.uk/resources/94_fractional-and-negative-indicesans.pdf

4. **Watch the video, then answer all the exam questions and mark your answers.**
   https://www.youtube.com/watch?v=qkrQchGWcGI

5. **Connect 4.** This game can be printed off and played against someone else on your own.
   https://justmaths.co.uk/Worksheets/Number/Fractional%20and%20negative%20indices%20-%20connect%204.pdf

IV. Investigations

For each of the following you should carry out the investigations then read the notes. You need to keep a detailed summary of what methods/approaches you have tried and what you then changed each time.

1. Pick one of the investigations in the booklet and answer the questions. In order to gain full credit you need to extend your investigation further. Explain at each stage the decisions you have made and any changes in your approach. Your investigation will need a detailed summary.
   http://social.ocr.org.uk/files/ocr/Maths%20investigations.pdf

2. NRICH activities. Follow the instructions, can you extend your thinking further? Follow some of the links? Keep notes of what you are doing at each stage.
   https://nrich.maths.org/2929
   https://nrich.maths.org/12558

https://nrich.maths.org/10124
https://nrich.maths.org/11726

3. Exploring maths. What's the longest chain you can make? Is there a strategy?
   https://wild.maths.org/factors-and-multiples-chain

4. **LCM in the real world.** Read and make notes. Can you find other examples of where we use LCM in the real world?
   http://madhuriesingh.com/worksheets-projects-craft-activities/where-in-real-life-we-use-lcm/

5. **Write a report.** What are these prime numbers? Can you investigate them further?
   https://en.wikipedia.org/wiki/Mersenne_prime

V. Academic Reading

1. **Graph theory.** Explore graph theory and the maths behind it. Prepare a report explaining the principals and maths involved. Use this link as a starting point.
   https://wild.maths.org/can-you-traverse-it

2. **Pigeons can do maths.** Read the article and make notes, there is a video to watch and links to follow to extend your understanding. You should think about ways you could present your findings.
   https://ibmathsresources.com/2017/02/10/even-pigeons-can-do-maths-2/

Maths in the real world.
For each video or article, you should make notes and questions you would like answering to extend your understanding and knowledge of maths in the real world.

3. **Follow the 'WATCH, THINK, DIG DEEPER, DISCUSS'**

The Rubix Cube
https://ed.ted.com/featured/VvOg8aiS

Follow the 'WATCH, THINK, DIG DEEPER, DISCUSS'
Can you find the next number?
https://ed.ted.com/lessons/can-you-find-the-next-number-in-this-sequence-alex-gendler

Follow the 'WATCH, THINK, DIG DEEPER, DISCUSS'
Cake!
https://ed.ted.com/featured/KTil2Gft

4. Read the following article, can you produce a poster, or presentation using this article? **Should maths be taught to all students until they are 18?** What do you think? Can you research arguments for and against?
   https://www.theguardian.com/commentisfree/2012/jul/25/compulsory-maths-lessons-until-18
   https://sfh10.wordpress.com/2010/10/25/should-maths-in-secondary-schools-be-compulsory/

5. **Mental multiplication tricks.**
   Watch the video then design instructions to explain how these methods work.
   https://www.youtube.com/watch?v=30X37X1uk2o

6. Pick one subject from the list and follow the materials and questions to extend your understanding about mathematics in other subjects.
   https://integralmaths.org/course/view.php?id=166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