

PiXL Independence:

Mathematics – Answer Booklet

KS4 HIGHER

Topic 3 - Factorising, Inequalities, Quadratics

Contents:

Answers

I. Basic Skills Check

Answer the following questions. In order to improve your basic arithmetic you should attempt these without a calculator where possible.

Skills Check 1

1. A coat is reduced by 12% to a price of £59.84. Calculate the original cost of the coat.

£68



2. What is $\frac{2}{3}$ of £126?

84

3. Write 58 as a product of its prime factors.

2 x 29

4. Factorise $x^2 - 6x - 16$.

$$(x-8)(x+2)$$

5. Write 760,000,000 in standard form.

7.6×10^8

6. Solve: $4x + 7 \leq -13$

$x \leq -5$

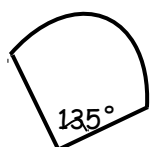
7. Use prime factors to find the lowest common multiple of 70 and 80.

560

8. List the first 5 terms of the sequence $-3n - 2$.

-5, -8, -11, -14, -17

9. Find the total perimeter of the sector shown, correct to one decimal place.



26.1cm



10. Calculate $(1.25 \times 10^{15}) \div (4.2 \times 10^9)$, giving your answer in standard form correct to **two** significant figures.

3.0×10^5

Skills Check 2

1. The height of a student is measured to the nearest cm, if it is recorded as 132cm what is the maximum and minimum height of the student? UB= 132.5 LB=131.5

2. Calculate $(6.1 \times 10^{12}) \times (2.4 \times 10^3)$, giving your answer in standard form correct to two significant figures.

$$1.5 \times 10^{16}$$

3. Write 40 as a product of prime factors. What is the LCM of 40 and 52?

$$2^3 \times 5 = 40$$

$$2^2 \times 13 = 52$$

$$\text{LCM} = 520$$

4. Factorise $x^2 - 2x - 80$

$$(x-10)(x+8).$$

5. Write 0.00000302 in standard form.

$$3.02 \times 10^{-6}$$

6. Solve: $-17 \leq 4 - 3x$

$$x \leq 7$$

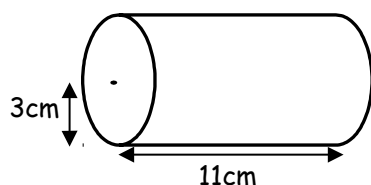
7. Calculate the total surface area of a cylinder with radius 3cm and length 11cm. Give your **final** answer to one decimal place.



$$\text{Circle area} = 18\pi$$

$$\text{Curved surface} = 66\pi$$

$$\text{Total} = 263.9\text{cm}^2$$



8. Find the n th term of the sequence: -2, 1, 6, 13..... $n^2 - 3$

9. Find the reciprocal of the number 3.6, giving your answer as a fraction.

$$5/18$$

10. The masses of a group of pupils are displayed in this table. Calculate an estimate of the mean mass.

Mass (x kg)	Frequency	MP	MP \times f
$40 \leq x < 50$	4	45	180
$50 \leq x < 60$	8	55	440
$60 \leq x < 70$	5	65	325
$70 \leq x < 80$	3	75	225

$$1170/20 = 58.5 \text{ kg}$$

Skills Check 3

1. Find the lower bound for the perimeter of this parallelogram if the measurements shown are correct to the nearest centimetre.



$$\begin{aligned} \text{LB} &= 9.5 \text{ and } 15.5 \\ &= 50\text{cm} \end{aligned}$$

2. Work out $\frac{5}{6} + \frac{3}{4}$, simplifying your answer as far as possible. $\frac{19}{12} = 1\frac{7}{12}$

3. Use prime factors to find the lowest common multiple of 112 and 84.

$$\text{LCM} = 336$$

4. Vicki rolls a dice 20 times. Her scores are recorded in this table. Calculate the mode, median and mean of her scores.

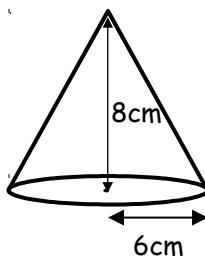
Score	1	2	3	4	5	6
Frequency	4	4	0	5	6	1

$$\text{Mode} = 5$$

$$\text{Median} = 4$$

$$\text{Mean} = 3.4$$

5. Calculate the total surface area of a cone with radius 6cm and vertical height 8cm. Give your answer as a single multiple of π .



$$\text{Curved surface} = 60\pi$$

$$\text{Base} = 36\pi$$

$$\text{Total} = 96\pi$$

6. Work out $(3 \times 10^{14}) \times (4 \times 10^5)$, giving your answer in standard form.

$$1.2 \times 10^{20}$$

7. Work out $295.05 \div 7$

42.15

8. Find the n th term of the sequence: 3, 12, 27, 48... $3n^2$



9. Use prime factors to find the highest common factor of 150 and 900.

150

10. Write the recurring decimal $0.101010101\dots$ as a fraction in its simplest form

$$x = 0.101010\dots$$

$$100x = 10.101010\dots$$

$$99x = 10$$

$$x = \frac{10}{99}$$

II. Short Exam Questions

Section 1 - Factorising and Simplifying

1. Expand and simplify each of these expressions:

a. $3(5x^2 - x + 4)$

$$15x^2 - 3x + 12$$

b. $2x(4x - 5)$

$$8x^2 - 10x$$

c. $4(x + 7) - 3(x - 2)$

$$4x + 28 - 3x + 6 = x + 34$$

d. $(x + 7)(x + 2)$

$$x^2 + 9x + 14$$

e. $(4x - 1)(2x + 5)$

$$8x^2 + 20x - 2x - 5 = 8x^2 + 18x - 5$$

f. $(5x - 3)^2$

$$25x^2 - 30x + 9$$

2. Factorise each of these expressions by removing common factors:

a. $3xy + 5y$

$$y(3x + 5)$$

b. $12x^3 - 18x^2$

$$6x^2(2x - 3)$$

c. $8xy + 4y$

$$4y(2x + 1)$$

3. In an exam, Robert factorises the expression $18y^3 - 36y^2$ to give the answer $9y(2y^2 - 4y)$. Explain why he would **not** receive full marks. This has not FULLY been factorised. It should be $18y^2(y - 2)$

4. Factorise each of these quadratic expressions using double brackets:

a. $x^2 + 9x + 14$ $(x+7)(x+2)$

b. $x^2 + 3x - 70$ $(x-7)(x+10)$

c. $x^2 - x - 90$ $(x-10)(x+9)$

d. $x^2 - 11x + 30$ $(x-6)(x-5)$

5. Factorise each of these harder quadratic expressions:

a. $2x^2 + 11x + 5$ $(2x+1)(x+5)$

b. $3x^2 - 10x + 7$ $(3x-7)(x-1)$

c. $5x^2 + 13x - 6$ $(5x-2)(x+3)$

6. Factorise each of these using the difference of two squares:

a. $y^2 - 36$ $(y-6)(y+6)$

b. $49y^2 - 81$ $(7y+9)(7y-9)$

c. $x^2 - 9y^2$ $(x+3y)(x-3y)$

7. Factorise each of these expressions **fully**:

a. $5y^2 + 30y + 40$ $5(x+2)(x+4)$

b. $y^3 - 16y$ $y(y^2 - 16)$

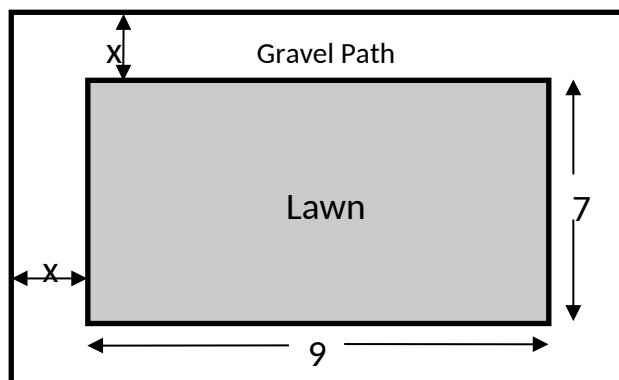
c. $6y^3 - 36y^2 + 54y$ $6y(y-3)^2$

8. Use factorisation to work out $53^2 - 47^2$ **without** a calculator.
Show every step in your working.

$(53+47)(53-47)=$

$100 \times 6 = 600$

9. Anna's back garden consists of a rectangular lawn measuring 9 metres by 7 metres, surrounded by a gravel path of width x metres. Find, and simplify, an expression for the total area of the garden.



$$\begin{aligned}\text{Path} &= (7+2x)(9+2x) \\ &= 63+14x+18x+4x^2 \\ &= 4x^2+32x+63\end{aligned}$$

Section 2 - Solving Equations

1. Solve each of these equations.

a) $4x - 3 = 15$

$x = 4.4$

b) $\frac{y}{3} + 4 = 9$

$y = 15$

c) $5m - 8 = 2m + 13$

$m = 7$

d) $24 - x = 3x + 16$

$x = 2$

e) $3(2k + 1) = 5(k + 2)$

$k = 7$

f) $3(m + 2) - 2(m - 4) = 10$

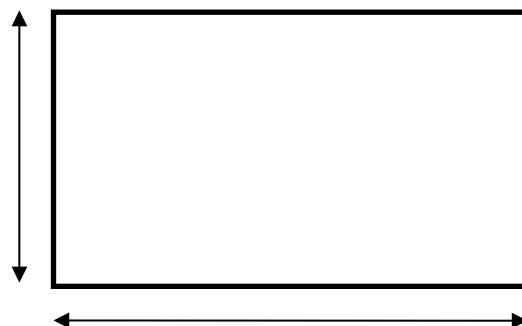
$m = -4$

2. The perimeter of the rectangle shown opposite is 38cm.

What is the value of x .

$2(3+2x)+2(4x-5) = 38$

7



$$6+4x+8x-10 = 38$$

$$12x = 42$$

$$x = 3.5$$

3. Solve each of these equations.

a) $\frac{d+3}{4} = 12$

$$d = 45$$

b) $\frac{2y+1}{3} = \frac{5y-3}{2}$

$$y = 1$$

c) $\frac{2m}{9} + \frac{m}{6} = 7$

$$m = 18$$

4. Solve the equation $\frac{x-2}{4} + \frac{x+3}{11} = 10$.

$$\frac{11x-22+4x+12}{44} = 10$$

$$\frac{15x-10}{44} = 10$$

$$15x = 450$$

$$x = 30$$

5. Simplify each of these algebraic fractions as far as possible.

a) $\frac{4h}{12}$

$$\frac{h}{3}$$

b) $\frac{5c}{20c^2}$

$$\frac{1}{4c}$$

c) $\frac{15y-21}{12}$

$$\frac{5y-7}{4}$$

d) $\frac{x^2+7x+10}{6x+12}$

$$\frac{(x+5)(x+2)}{6(x+2)} = \frac{(x+5)}{6}$$

$$\text{e) } \frac{x^2 - 16}{x^2 + 4x}$$

$$\frac{(x+4)(x-4)}{x(x+4)} = \frac{(x-4)}{x}$$

6. Simplify these fractional multiplications and divisions.

$$\text{a) } \frac{y}{4} \div \frac{y}{3}$$

$$\frac{3}{4}$$

$$\text{b) } \frac{4m}{5p^2} \times \frac{15p^2}{8m^3}$$

$$\frac{3}{2m^2}$$

$$\text{c) } \frac{x+1}{3} \div \frac{x^2+5x+4}{9}$$

$$\frac{x+1}{3} \times \frac{9}{x^2+5x+4} = \frac{9(x+1)}{3(x+1)(x+4)} = \frac{3}{x+4}$$

7. Write each of these expressions as a single algebraic fraction. Simplify your answers where appropriate.

$$\text{a) } \frac{3y}{4} - \frac{y}{6}$$

$$\frac{7y}{12}$$

$$\text{b) } \frac{x+4}{5} + \frac{x-1}{6}$$

$$\frac{6(x+4)+5(x-1)}{30} = \frac{6x+24+5x-5}{30} = \frac{11x+19}{30}$$

$$\text{c) } \frac{1}{x-3} + \frac{2}{x+4}$$

$$\frac{x+4+2x-6}{(x-3)(x+4)} = \frac{3x-2}{(x-3)(x+4)}$$

8. You are given the equation $\frac{120}{x-3} + \frac{180}{x+4} = 15$.

Setting out each stage of your working clearly, show that this equation can be transformed into the form

$$x^2 - 19x - 8 = 0$$

$$120x + 480 + 180x - 540 = 15(x-3)(x+4)$$

$$300x - 60 = 15(x^2 + x - 12)$$

$$20x - 4 = x^2 + x - 12$$

Which gives $x^2 - 19x - 8 = 0$ as required

Section 3 - Inequalities and Simultaneous Equations

1. Solve these inequalities and represent the solutions on a number line.

a) $3x < 24$ $x < 8$

b) $2x - 5 > 17$ $x > 11$

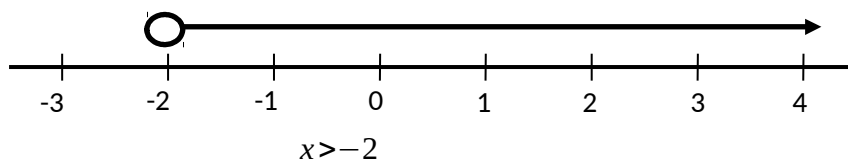
c) $2(x + 5) \leq 16$ $x \leq 3$

d) $7x - 5 \geq 3x + 3$ $x \geq 2$

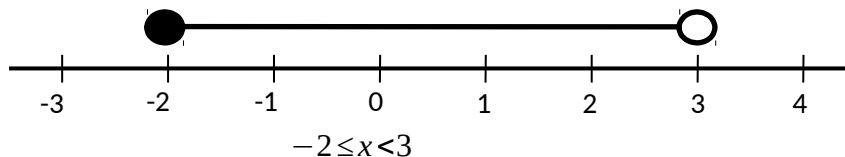
e) $3x + 1 < x + 3$ $x < 1$

2. Write down inequalities to describe each of these number lines.

b)



c)



3. Write down all the integer values of x included in the inequality $-4 \leq x < 3$.
-4, -3, -2, -1, 0, 1, 2

4. Write down all the integer values of x included in the inequality $-4 \leq 2x < 3$.
-2, -1, 0, 1

5. Tickets to a fair cost £4.75 for adults and £2.50 for children. A coach party of 48 people arrives and their tickets cost a total of £147. Form a pair of simultaneous equations, and solve them to find the number of adults and children on this coach.

$4.75A + 2.50C = 147$

$A + C = 48$

$4.75A + 2.5C = 147$

$2.50A + 2.5C = 120$

$2.25A = 27$

Adults = 12

Children = 36

6. The length of a rectangle is 6cm more than its width.
The area of the rectangle is 55cm².

a. Form a quadratic equation to represent this information.

$$w(w+6)=55$$

$$w^2+6w=55$$

$$w^2+6w-55=0$$

$$(w+11)(w-5)=0$$

$$w=-11 \text{ or } 5$$

Since length it can't be -11. $w = 5$

b. Solve your equation to find the dimensions of this rectangle.

Width = 5, Length = 11

7. Solve these simultaneous equations algebraically:

$$7x+6y=9$$

$$8x-15y=54$$

Multiplying to get equal numbers of y gives:

$$35x+30y=45$$

$$16x-30y=108$$

Adding these gives $51x=153$ so $x=3$

Substituting into the first equation gives

$$21+6y=9$$

$$6y=-12$$

$$y=-2$$

8. Bobby buys 28 tins of beans. Large tins cost 73p and small tins cost 49p. Altogether his beans cost £17.80.

How many large tins and how many small tins did he buy?

The question leads to these simultaneous equations:

$$L+S=28$$

$$73L+49S=1780$$

Multiplying the first equation by 49:

$$49L+49S=1372$$

$$73L+49S=1780$$

Subtracting these gives:

$$24L=408$$

$$L=17$$

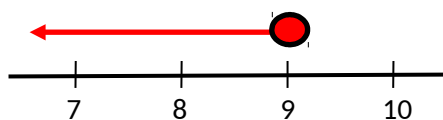
So Bobby buys 17 large and 11 small tins of beans

9. Solve the inequality $6x-3 \leq 4x+15$ and show your answer on a number line.

$$2x-3 \leq 15$$

$$2x \leq 18$$

$$x \leq 9$$



10. Solve these non-linear simultaneous equations:

$$\begin{aligned} 3x + y &= 19 \\ y &= x^2 + 5x - 1 \end{aligned}$$

ANS $3x + (x^2 + 5x - 1) = 19$

$$x^2 + 8x - 20 = 0$$

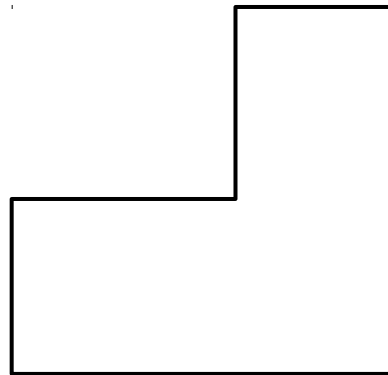
$$(x + 10)(x - 2) = 0$$

$$x = -10, \quad x = 2$$

Solutions: $x = -10$ with $y = 49$ and $x = 2$ with $y = 13$.

Section 4 - Mixed Questions

1. Find and simplify expressions for the perimeter and area of this compound shape.



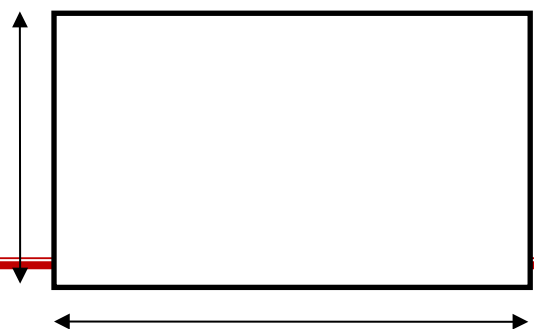
Perimeter = total of all six sides = $3x + 4x + 1 + 3x + 6 + x + 7 + 6 + 3x - 6 = 14x + 14$

or $14(x + 1)$

Area = $3x(4x + 1) + 6(x + 7) = 12x^2 + 9x + 42$ or $3(4x^2 + 3x + 14)$

2. The perimeter of the rectangle shown opposite is 32cm. Find the value of x and the area of the rectangle.

12



$$2(x-3)+2(2x+1)=32 \quad \text{so} \quad 2x-6+4x+2=32 \quad \text{so} \quad 6x-4=32 \quad \text{so} \quad x=6$$

Length = 13cm, width = 3cm so area = 39cm²

3. Ben cycles from Acton to Beeswell and back again, a distance of 60 miles in each direction. On the outward journey he averages x mph but on the return journey his average is 3mph less. The total journey takes 12 hours. Write this

information as an equation, and then show that it can be rearranged to make $x^2 - 13x + 15 = 0$

(Remember that time taken = $\frac{\text{distance}}{\text{speed}}$)

$$\frac{60}{x} + \frac{60}{x-3} = 12$$

Question leads to . Cross-multiplying gives

$$60x - 180 + 60x = 12x(x-3) \quad \text{which leads to} \quad 0 = 12x^2 - 156x + 180 \quad \text{. Dividing}$$

$$\text{by 12 gives} \quad x^2 - 13x + 15 = 0 \quad \text{, as required.}$$

4. Simplify as far as possible

a) $\frac{3A}{8B^3} \div \frac{15A^2}{16B^2}$

b) $\frac{x-2}{6} + \frac{3-x}{9}$

c) $\frac{2}{x+5} + \frac{7}{x-4}$

ANS: $\frac{3A}{8B^3} \times \frac{16B^2}{15A^2} = \frac{2}{5AB}$

b. $\frac{9(x-2)+6(3-x)}{54} = \frac{3x}{54} = \frac{x}{18}$

$$c. \quad \frac{2x-8+7x+35}{(x+5)(x-4)} = \frac{9x+27}{(x+5)(x-4)} \quad \text{or} \quad \frac{9(x+3)}{(x+5)(x-4)}$$

5. Simplify as far as possible;

$$a. \quad \frac{12k^2d^8}{3k^2d^3} = 4d^5 \qquad b. \quad (3d^2h)^3 = 27d^6h^3$$

6. Expand and simplify the expression $(2x-5)(x+3)(x+4)$.

$$\text{ANS} \quad (2x^2+6x-5x-15)(x+4) = (2x^2+x-15)(x+4) = \\ 2x^3+x^2-15x+8x^2+4x-60 = 2x^3+9x^2-11x-60$$

7. Natalie is 'a' years old. Write down expressions in terms of a for the following people's ages:

- a) Joyce, who is 10 years older than Natalie $a+10$
 b) John, who is half Natalie's age. $a/2$
 c) Gavin, who is twice Joyce's age. $2(a+10)$
 d) Steven, who is 4 years older than John. $\frac{a}{2}+4$

8. The angles in a triangle are x, 3x and 5x.

Write an equation to find the value of x.

$$x+3x+5x=180$$

$$9x=180$$

$$x=20$$

Write down the size of each angle in the triangle.

$$20^\circ, 60^\circ, 100^\circ$$

9. The four angles of a quadrilateral are 45° , 105° , $(4x-15)^\circ$ and $5x^\circ$.

- a) Form an equation, in terms of x, using this information.

$$45+105+4x-15+5x=360$$

$$135+9x=360$$

$$x=25$$

- b) Solve your equation and work out the size of the largest angle of the quadrilateral.

$$\text{Angles} = 45, 105, 85, 100 \text{ so largest angle} = 100^\circ$$

10. The length of a rectangle is 5cm more than its width. The area of the rectangle is 18cm^2 . Form and solve a quadratic equation to find the width of this rectangle. Give your answer correct to two decimal places.

$$w(w+5)=18$$

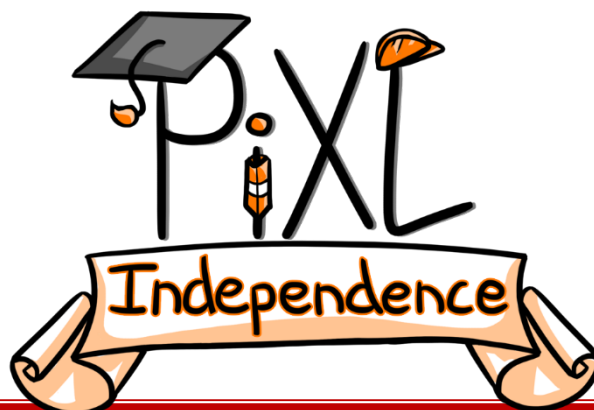
$$w^2+5w-18=0$$

$$w = \frac{-5 \pm \sqrt{25 - 4 \times 1 \times -18}}{2}$$

(Watch minus signs here!)

$$w = \frac{-5 \pm \sqrt{25+72}}{2}$$

Width = 2.42cm (or -7.42cm)



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