PiXL Independence:

Mathematics - Student Booklet
KS4 HIGHER

Topic 3 – Equations, Inequalities and Quadratics

Contents:

I. Basic Skills Check – 10 credits per skills check.

II. Short Exam Questions - 30 credits per section.

III. Further Practice – 30 credits each.

IV. Investigations – 80 credits each.

V. Academic Stretch – 50 credits each.

I.   Basic Skills Check

Answer the following questions. In order to improve your basic arithmetic you should attempt these without a calculator where possible.

Skills Check 1

1. A coat is reduced by 12% to a price of £59.84. Calculate the original cost of the coat.

2. What is $\frac{2}{3}$ of £126?

3. Write 58 as a product of its prime factors.
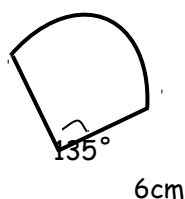
4. Factorise x^2 -6x -16.

5. Write 760,000,000 in standard form.

6. Solve: $4x+7 \leq -13$.

7. Use prime factors to find the lowest common multiple of 70 and 80.

8. List the first 5 terms of the sequence -3n- 2.

9. Find the total perimeter of the sector shown, correct to one decimal place.

135°

6cm

10. Calculate $(1.25 \times 10^{15}) \div (4.2 \times 10^{9})$, giving your answer in standard form correct to **two** significant figures.

Skills Check 2

1. The height of a student is measured to the nearest cm, if it is recorded as 132cm what is the maximum and minimum height of the student?

2. Calculate $\left(6.1\times10^{12}\right)\times\left(2.4\times10^{3}\right)$, giving your answer in standard form correct to two significant figures.

3. Write 40 as a product of prime factors. What is the LCM of 40 and 52?

4. Factorise x^2 -2x – 80.

5. Write 0.00000302 in standard form.

6. Solve: $-17\leq4-3x$.

7. Calculate the total surface area of a cylinder with radius 3cm and length 11cm. Give your **final** answer to one decimal place.

3cm

11cm
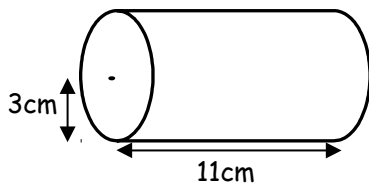
8. Find the nth term of the sequence: -2, 1, 6, 13…..

9. Find the reciprocal of the number 3.6, giving your answer as a fraction.

10. The masses of a group of pupils are displayed in this table. Calculate an estimate of the mean mass.

| Mass (x kg)	Frequency
$40 \leq x < 50$	4
$50 \leq x < 60$	8
$60 \leq x < 70$	5
$70 \leq x < 80$	3

Skills Check 3

1. Find the lower bound for the perimeter of this parallelogram if the measurements shown are correct to the nearest centimetre.

10cm

16cm
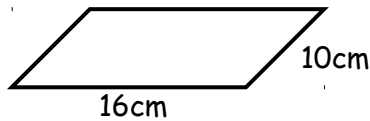
2. Work out $\dfrac{5}{6} + \dfrac{3}{4}$, simplifying your answer as far as possible.

3. Use prime factors to find the lowest common multiple of 112 and 84.

4. Vicki rolls a dice 20 times. Her scores are recorded in this table. Calculate the mode, median and mean of her scores.

Score	1	2	3	4	5	6
Frequency	4	4	0	5	6	1

5. Calculate the total surface area of a cone with radius 6cm and vertical height 8cm. Give your answer as a single multiple of π.

8cm

6cm
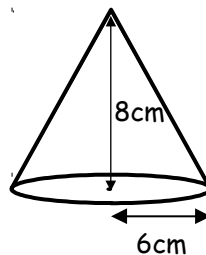
6. Work out $\left(3 \times 10^{14}\right) \times \left(4 \times 10^{5}\right)$, giving your answer in standard form.

7. Work out 295.05 ÷ 7

8. Find the nth term of the sequence: 3, 12, 27, 48....

9. Use prime factors to find the highest common factor of 150 and 900.

10. Write the recurring decimal $0.101010101....$ as a fraction in its simplest form.

Section 1 – Factorising and Simplifying

1.   Expand and simplify each of these expressions:

   a. $3(5x^2-x+4)$

   b. $2x(4x-5)$

   c. $4(x+7)-3(x-2)$

   d. $(x+7)(x+2)$

   e. $(4x-1)(2x+5)$

   f. $(5x-3)^2$

2.   Factorise each of these expressions by removing common factors:

   a. $3xy+5y$

   b. $12x^3-18x^2$

   c. $8xy+4y$

3.   In an exam, Robert factorises the expression $18y^3-36y^2$ to give the answer $9y(2y^2-4y)$. Explain why he would **not** receive full marks.

4.   Factorise each of these quadratic expressions using double brackets:

   a. $x^2+9x+14$

   b. $x^2+3x-70$

   c. x^2-x-90

   d. $x^2-11x+30$

5.   Factorise each of these harder quadratic expressions:

   a. $2x^2+11x+5$

   b. $3x^2-10x+7$

c. $5x^2 + 13x - 6$

6. Factorise each of these using the difference of two squares:

a. $y^2 - 36$

b. $49y^2 - 81$

c. $x^2 - 9y^2$

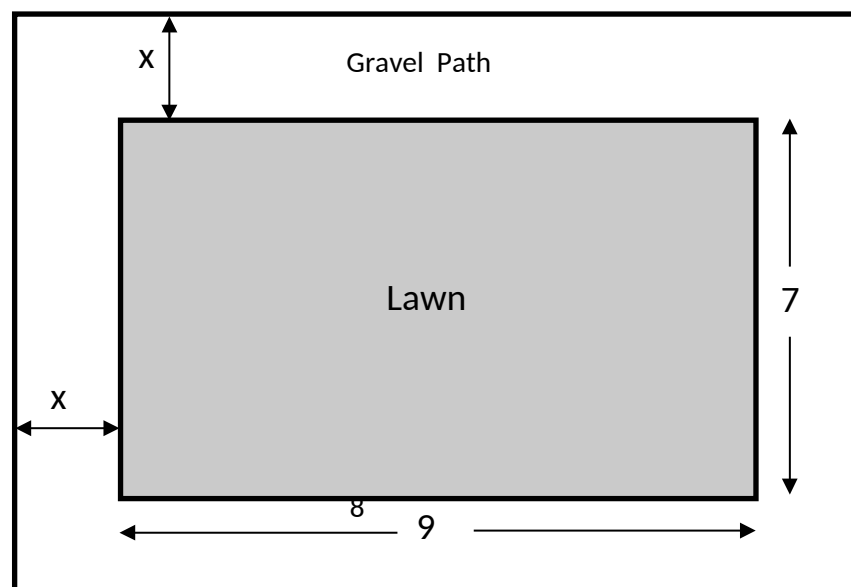7. Factorise each of these expressions **fully**:

a. $5y^2 + 30y + 40$

b. $y^3 - 16y$

c. $6y^3 - 36y^2 + 54y$

8. Use factorisation to work out $53^2 - 47^2$ **without** a calculator.
   Show every step in your working.

9. Anna's back garden consists of a rectangular lawn measuring 9 metres by 7 metres,
   surrounded by a gravel path of width x metres.
   Find, and simplify, an expression for the total area of the garden.

Section 2 – Solving Equations

1. Solve each of these equations.

   a) $4x - 3 = 15$

   b) $\dfrac{y}{3} + 4 = 9$
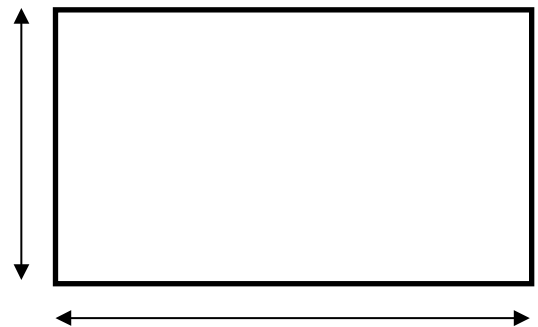
   c) $5m - 8 = 2m + 13$

   d) $24 - x = 3x + 16$

   e) $3(2k + 1) = 5(k + 2)$

   f) $3(m + 2) - 2(m - 4) = 10$

2. The perimeter of the rectangle shown opposite is 38cm.

   Find the value of x.

3. Solve each of these equations.

   a) $\dfrac{d + 3}{4} = 12$

b) $\dfrac{2y+1}{3} = \dfrac{5y-3}{2}$

c) $\dfrac{2m}{9} + \dfrac{m}{6} = 7$

4. Solve the equation $\dfrac{x-2}{4} + \dfrac{x+3}{11} = 10$.

5. Simplify each of these algebraic fractions as far as possible.

a) $\dfrac{4h}{12}$

b) $\dfrac{5c}{20c^2}$

c) $\dfrac{15y-21}{12}$

d) $\dfrac{x^2+7x+10}{6x+12}$

e) $\dfrac{x^2-16}{x^2+4x}$

6. Simplify these fractional multiplications and divisions.

a) $\dfrac{y}{4} \div \dfrac{y}{3}$

b) $\dfrac{4m}{5p^2} \times \dfrac{15p^2}{8m^3}$

c) $\dfrac{x+1}{3} \div \dfrac{x^2+5x+4}{9}$

7. Write each of these expressions as a single algebraic fraction. Simplify your answers where appropriate.

a) $\dfrac{3y}{4}-\dfrac{y}{6}$

b) $\dfrac{x+4}{5}+\dfrac{x-1}{6}$

c) $\dfrac{1}{x-3}+\dfrac{2}{x+4}$

8. You are given the equation $\dfrac{120}{x-3}+\dfrac{180}{x+4}=15$.
Setting out each stage of your working clearly, show that this equation can be transformed into the form $x^2-19x-8=0$.

Section 3 – Inequalities and Simultaneous Equations

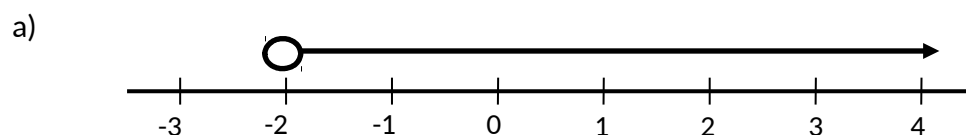1. Solve these inequalities and represent the solutions on a number line.

   a) 3x < 24
   b) 2x – 5 > 17
   c) 2(x + 5) ≤ 16
   d) 7x – 5 ≥ 3x + 3
   e) 3x + 1 < x + 3

2. Write down inequalities to describe each of these number lines.

   a)

   b)

11

3. Write down all the integer values of x included in the inequality $-4 \leq x < 3$.

4. Write down all the integer values of x included in the inequality $-4 \leq 2x < 3$.

5. Tickets to a fair cost £4.75 for adults and £2.50 for children. A coach party of 48 people arrives and their tickets cost a total of £147. Form a pair of simultaneous equations, and solve them to find the number of adults and children on this coach.

6. The length of a rectangle is 6cm more than its width.
   The area of the rectangle is 55cm².

   a. Form a quadratic equation to represent this information.
   b. Solve your equation to find the dimensions of this rectangle.
      (Both its length **and** its width)

7. Solve these simultaneous equations algebraically:

   $$7x + 6y = 9$$
   $$8x - 15y = 54$$

8. Bobby buys 28 tins of beans. Large tins cost 73p and small tins cost 49p. Altogether his beans cost £17.80.

   How many large tins and how many small tins did he buy?

9. Solve the inequality $6x - 3 \leq 4x + 15$ and show your answer on a number line.

10. Solve these non-linear simultaneous equations:

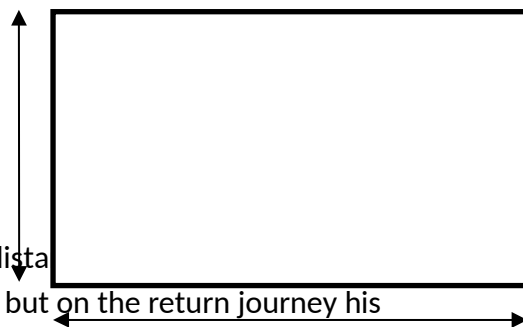 $$3x + y = 19$$
   $$y = x^2 + 5x - 1$$

Section 4 – Mixed Questions

1. Find and simplify expressions for the **perimeter** and **area** of
    this compound shape.
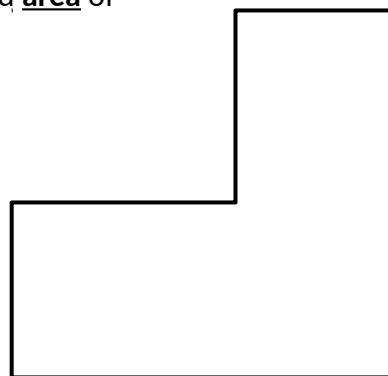
2. The perimeter of the rectangle shown opposite is 32cm.

Find the value of *x* and the **area** of the rectangle.

3. Ben cycles from Acton to Beeswell and back again, a dista[ ] direction. On the outward journey he averages x mph but on the return journey his average is 3mph **less**. The total journey takes 12 hours. Write this information as an equation, and then show that it can be rearranged to make $x^2 - 13x + 15 = 0$.

4. Simplify as far as possible

   a) $\dfrac{3A}{8B^3} \div \dfrac{15A^2}{16B^2}$

   b) $\dfrac{x-2}{6} + \dfrac{3-x}{9}$

   c) $\dfrac{2}{x+5} + \dfrac{7}{x\text{-}4}$

5. Simplify as far as possible;

   a) $\dfrac{12k^2 d^8}{3k^2 d^3}$ b) $\left(3d^2 h\right)^3$

6. Expand and simplify the expression $(2x-5)(x+3)(x+4)$.

7. Natalie is 'a' years old. Write down expressions in terms of **a** for the following people's ages:
   a) Joyce, who is 10 years older than Natalie.

b) John, who is half Naomi's age.
c) Gavin, who is twice *Joyce's* age.
d) Steven, who is 4 years older than *John*.

8. The angles in a triangle are x, 3x and 5x.
<u>Write an equation</u> to find the value of x.
Write down the size of each angle in the triangle.

9. The four angles of a quadrilateral are 45°, 105°, $(4x - 15)$° and $5x$°.
   a) Form an equation, in terms of x, using this information.

   b) Solve your equation and work out the size of the largest angle of the quadrilateral.

10. The length of a rectangle is 5cm more than its width. The area of the rectangle is 18cm^2. Form and solve a quadratic equation to find the width of this rectangle. Give your answer correct to two decimal places.

III.   Further Practice

1. **Follow the 'revise, activity, test'.** For each of the three topics the links to the 'revise' part are below. You might want to take screen shots, or make notes as you go to demonstrate how you have got on. Then carry out the activity and the test. Record your score, or try and improve. Check with your teacher which exam board you

are using, you might want to change the exam board on the home page.

https://www.bbc.co.uk/education/guides/z8y9jty/revision
https://www.bbc.co.uk/education/guides/z8y9jty/test

https://www.bbc.co.uk/education/guides/zpjhy4j/revision
https://www.bbc.co.uk/education/guides/zpjhy4j/test

https://www.bbc.co.uk/education/guides/z9vkqhv/revision
https://www.bbc.co.uk/education/guides/z9vkqhv/test

d) **Exam style practice.** For each of the topics you should watch the video, then answer the exam questions and mark your answers. Where have you made mistakes? Is there something you need to do more work on?

Algebraic fractions.
http://www.mathsgenie.co.uk/algebraic-fractions.html
http://www.mathsgenie.co.uk/resources/algebraic-fractions.pdf
http://www.mathsgenie.co.uk/resources/algebraic-fractionsans.pdf

Forming equations.
http://www.mathsgenie.co.uk/forming-and-solving-equations.html
http://www.mathsgenie.co.uk/resources/64_forming-and-solving-equations.pdf
http://www.mathsgenie.co.uk/resources/64_forming-and-solving-equationsans.pdf

Quadratic inequalities.
http://www.mathsgenie.co.uk/quadratic-inequalities.html
http://www.mathsgenie.co.uk/resources/quadraticinequalities.pdf
http://www.mathsgenie.co.uk/resources/quadraticinequalitiesans.pdf

e) **Watch the video, then answer all the exam questions and mark your answers.**
https://www.youtube.com/watch?v=9fnOtn6Q77Y

f) **Answer the exam style questions.**

https://justmaths.co.uk/Worksheets/Algebra/Simultaneous%20equations%20-%20EXAM%20QUESTIONS.pdf

g) **Answer the exam questions.** There are links on the front of the paper to videos you can access if you are still unsure.

https://corbettmaths.files.wordpress.com/2013/02/algebraic-fractions.pdf

https://corbettmaths.files.wordpress.com/2013/02/equations-fractional-pdf.pdf

h) **Worksheet with extension tasks.** Link to answers and extra videos for more support.

https://corbettmaths.files.wordpress.com/2013/02/equations-fractional-advanced-pdf1.pdf

https://corbettmaths.files.wordpress.com/2013/02/factorising-harder-quadratics-pdf1.pdf

https://corbettmaths.files.wordpress.com/2013/02/solving-inequalities-pdf.pdf

Answers: https://corbettmaths.com/contents/

IV.   Investigations

For each of the following you should carry out the investigations then read the notes. You need to keep a detailed summary of what methods/approaches you have tried and what you then changed each time.

1. Pick one of the investigations in the booklet and answer the questions. In order to gain full credit you need to extend your investigation further. Explain at each stage the decisions you have made and any changes in your approach. Your investigation will need a detailed summary.

http://social.ocr.org.uk/files/ocr/Maths%20investigations.pdf

2. NRICH activities. Follow the instructions, can you extend your thinking further? Follow some of the links. Keep notes of what you are doing at each stage.

https://nrich.maths.org/10196
https://nrich.maths.org/12925
https://nrich.maths.org/8678

3. **Exploring maths**. Can you follow the clues to match people? Then the extension question asks you to design your own version.

https://wild.maths.org/whos-who

4. **The problem of time.**

https://nrich.maths.org/528

V. Academic Reading

1. **Graph theory.** Explore graph theory and the maths behind it. Prepare a report explaining the principals and maths involved. Use this link as a starting point.
   https://wild.maths.org/can-you-traverse-it

2. **Pigeons can do maths.** Read the article and make notes, there is a video to watch and links to follow to extend your understanding. You should think about ways you could present your findings.
   https://ibmathsresources.com/2017/02/10/even-pigeons-can-do-maths-2/

Maths in the real world.

For each video or article you should make notes and questions you would like answering to extend your understanding and knowledge of maths in the real world.

3. **Follow the 'WATCH, THINK, DIG DEEPER, DISCUSS'**
   The fish riddle.
   https://ed.ted.com/lessons/can-you-solve-the-fish-riddle-steve-wyborney

   Follow the 'WATCH, THINK, DIG DEEPER, DISCUSS'
   How to squeeze electricity out of crystals.
   https://ed.ted.com/lessons/how-to-squeeze-electricity-out-of-crystals-ashwini-bharathula

   Follow the 'WATCH, THINK, DIG DEEPER, DISCUSS'
   https://ed.ted.com/lessons/a-brief-history-of-numerical-systems-alessandra-king

4. **Mathematical tricks and calculations. How do you do it?** Can you write detailed instructions on how to carry out these calculations?
   https://www.youtube.com/watch?v=hLdKsKep1og
   https://www.youtube.com/watch?v=PYrgjMubh-c

5. **Why study algebra?** Create a report to argue one side of the argument, is it really worth it?
   http://www.mathscareers.org.uk/article/10-reasons-for-studying-algebra/

6. Pick one subject from the list and follow the materials and questions to extend your understanding about mathematics in other subjects.

   https://integralmaths.org/course/view.php?id=166

7. **Articles, CRYPTOGRAPHY.** Read the following articles, make detailed notes, follow the links and extend your understanding further.
   https://nrich.maths.org/8032
   https://nrich.maths.org/2200
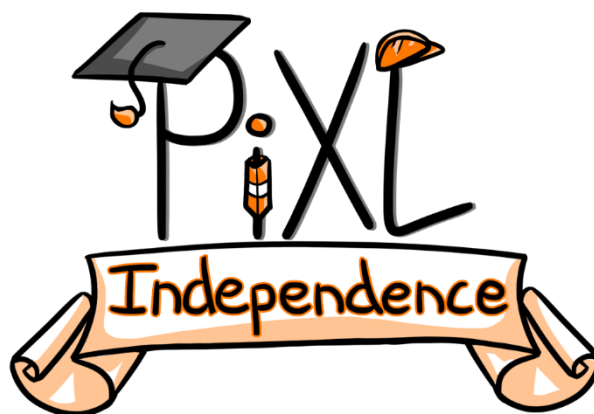
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